PHYS 393
Low Temperature Physics
Set 4:

Superconductivity – Theory

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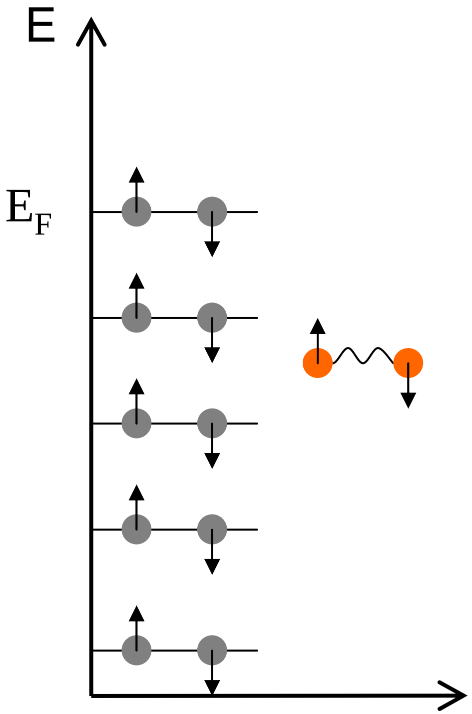
Superconductivity theory

- Many efforts over decades resulted in models with limited success.
- Breakthrough in 1957 (Bardeen, Cooper, Schrieffer), BCS theory:
  - Pairs of electrons (fermions) can couple to have zero or integer angular momentum and thus become bosons (Cooper pairs, 1956)
  - Pairing interaction mechanism (electron-phonon-electron) proposed in 1950 independently by Fröhlich (Liverpool) and Bardeen

Cooper pair formation

- Fermi sea: all states with $E<E_F$ (Fermi energy) occupied
- Addition of non-interacting electron pair will stay on surface (E_F)
- In the presence of small attractive interaction between pair they will "immerse" in Fermi sea as the attractive interaction lowers the pair's energy
- Question: what can create attractive interaction between two electrons (overcoming the strong Coulomb repulsion)?



Sketch of Fermi gas (electrons inside conductor) and a Cooper pair (right)

Pairing interaction

- Electron flying through lattice of positive ions exerts attractive force on them creating a positive cloud in its wake
- This surplus of positive charge attracts another electron
- Development of ion cloud delayed by time of order of lattice vibration period (10^{-13}s)
- Electrons move typically with 10^8cm/s
- Thus first electron has moved by 100nm before second electron is attracted; distance reduces Coulomb repulsion leaving net attractive force
- Electron-cloud-electron moves coupled through the material: Cooper pair
- Interaction described as exchange of virtual phonons

Cooper pairs

- Phonon exchange most efficient for vanishing center-of-mass momentum of the pair.
- Hence electrons have wavevectors $\underline{k}$ and $-\underline{k}$
- This corresponds to space wavefunction with zero angular momentum: $L=0$
- Spin wavefunction is antisymmetric (↑↓): $S=0$
- Overall wavefunction antisymmetric under particle exchange (made of fermions)
- But total angular momentum zero: $\underline{J}=\underline{L}+\underline{S}=0+0=0$
- Hence pair acts as a boson

BCS Ground State

- Many bosons can occupy the ground state
- Correlated wavefunction of pairs: single wavefunction for all as in Bose-Einstein condensate
- Identify superconducting current with that carried by Cooper pairs
- Normal current carried by single electrons in excited states
- Ground state separated from first excited state by energy gap Δ
- First excited state: broken pairs: for each pair we get two electron-hole pairs

Summary points

- Superconductivity depends on condensation of bosons (Cooper pairs) into ground state
- Critical temperature T_C set by strength of pairing interaction, not by B-E condensation ($T_B >> T_C$)
- Magnetic field destroys superconductivity as its action is to align electron spins, breaking up ↑↓ pairs
- BCS theory relates energy gap at T=0 $\Delta(0)$ to electron-lattice interaction strength V via:

$$\Delta(0) = 1.76kT_C = 2\hbar\omega_D e^{-\frac{1}{g(\mu)V}}$$

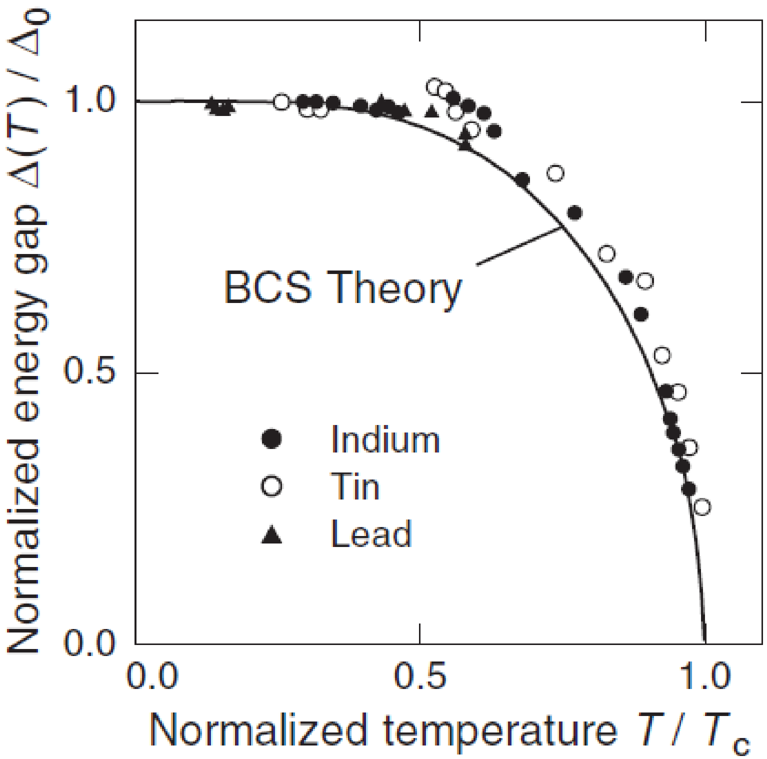
Interaction strength

Density of states at Fermi surface

Comments

- For metals the model factor 1.76 is in the range 1.6-2.15: reasonable agreement
- Energy gap Δ effectively constant in region $0<T<T_C/2$
- For $T_C/2<T<T_C$ it varies as:

$$\Delta(T) \simeq\sim 3.1kT_C\left[1-\frac{T}{T_C}\right]^{1/2}$$



On interaction strength

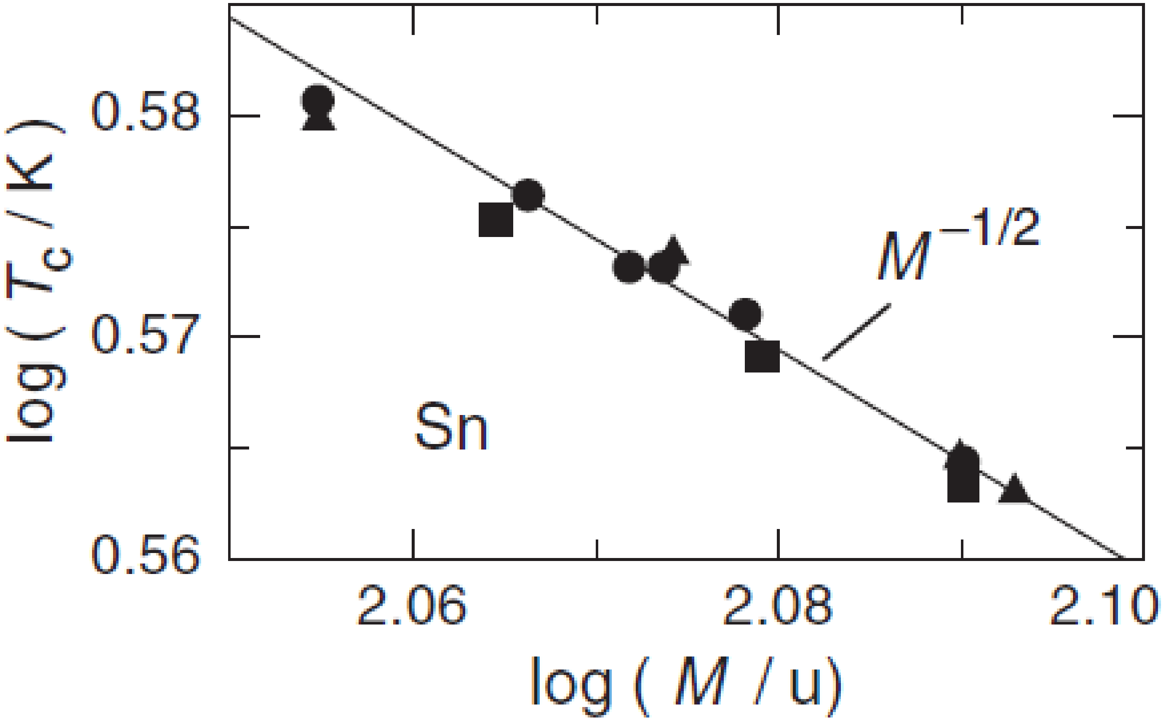
$$\Delta(0) = 1.76 kT_C = 2\hbar\omega_D e^{-\frac{1}{g(\mu)V}}$$

- Large electron-lattice interaction V related to high T_C
- In normal state large V means large resistivity
- This is the connection between superconductivity and poor metallic conductivity in normal state
- This is why good conductors do not show superconductivity

Isotope effect

- First evidence that lattice is involved in pairing interaction
- Measurements of T_C for different isotopes of same element versus isotope mass M:

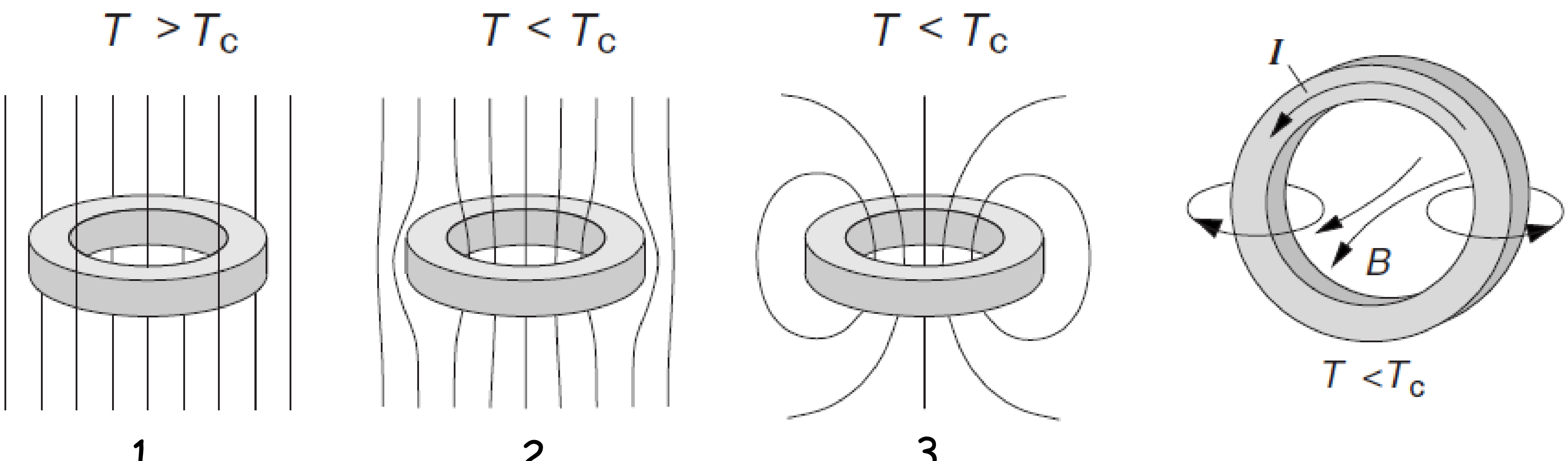
$$T_C \propto \sqrt{\frac{1}{M}}$$



Persistent currents

1. Magnetic field at $T>T_C$ (normal conducting state) penetrates material
2. Temperature lowered below T_C: magnetic field expelled from material (Meissner effect)
3. Magnetic field switched off: magnetic flux cannot be expelled from interior of ring as it would have to cross material: magnetic flux remains frozen

- Frozen flux ↔ persistent current



Superconducting ring with persistent current

Coherence and macroscopic wavefunction

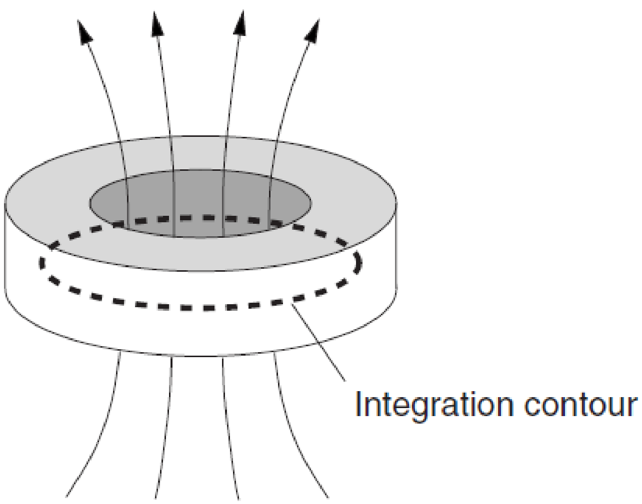
- Maximum energy uncertainty for Cooper pairs is 2Δ, leading to position uncertainty of order 1000nm (coherence length)
- Large than Cooper pair size
- Leading to large space overlaps
- Implying high correlation between pairs
- Indeed in B.E.C. (ground state) all Cooper pairs belong to common quantum state
- This is described by macroscopic wavefunction $\Psi=\Psi_0\exp(i\phi(r))$
- There is a well-defined phase in the whole sample

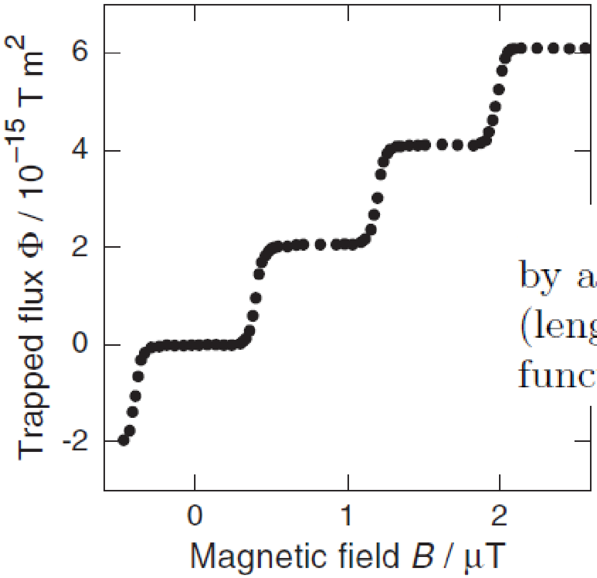
Flux quantization (I)

- Macroscopic wavefunction has unique phase
- Wavefunction is single-valued
- Phase change for a closed contour must be $2\pi n$
- Leading (after a bit of algebra) to:

$$\Phi = n\Phi_0 \quad \text{with flux quantum} \quad \Phi_0 = \frac{h}{2e}$$

In agreement with charge of current carrier (Cooper pair)





Magnetic flux trapped by a thin hollow cylinder made of tin (length 24 mm, diameter 56 µm) as a function of the cooling field

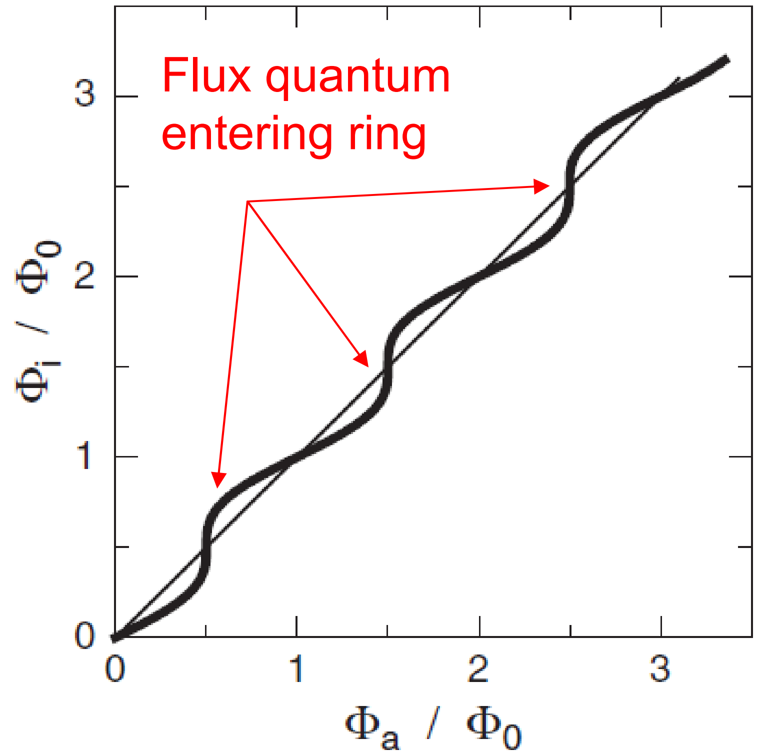
Flux quantization (II)

- Flux quantization $\Rightarrow$ current quantization
- Continuous variation of current is not allowed
- Phase change could be allowed, but this would temporarily destroy Cooper pair coherence, raising condensation energy
- Therefore no such jump can occur, and no flux quanta can leave the superconducting loop
- Hence persistent currents are absolutely stable

Josephson Effect

- Josephson junction: thin (<2nm) resistor layer between two superconductors (weak link): Cooper pairs can tunnel through junction allowing current flux
- Think of ring in previous case with a weak link at one point: flux can escape from ring, one line (quantum Φ_0) at a time.

Graph of variation of flux inside the ring Φ_i versus flux $\Phi_a = A \bullet B$ due to external field B if ring was not superconducting, where A is the ring surface:



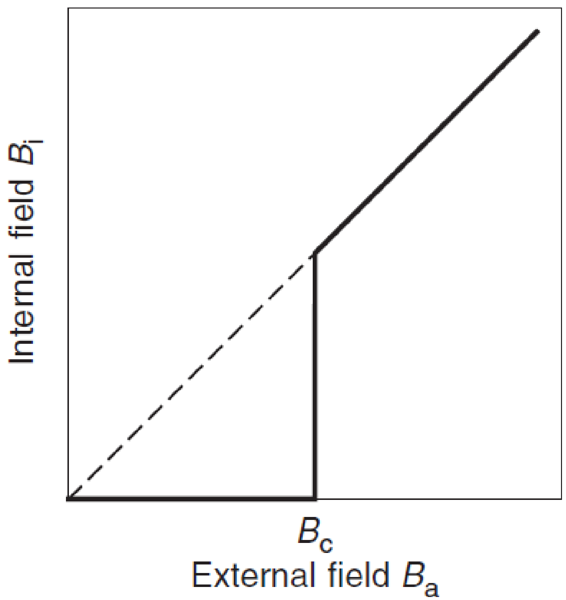
Superconducting QUantum Interference Device

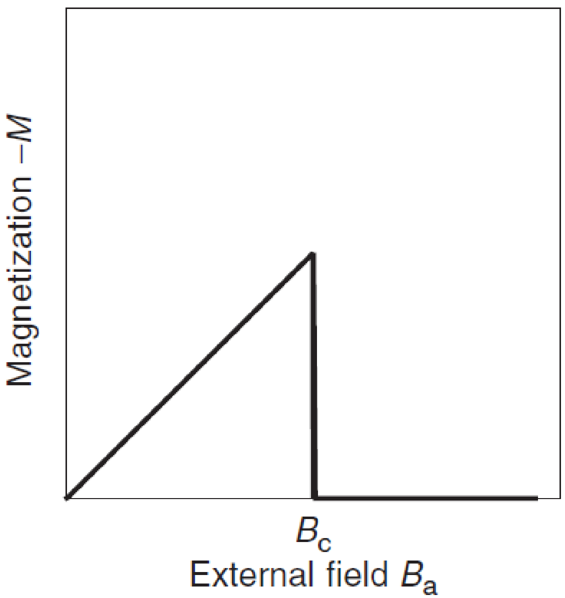
- SQUID: superconducting ring with one or two Josephson junctions coupled to sensitive measuring circuit
- Can measure magnetic field variations down to 10^{-14}T
- Most sensitive magnetic field measuring device
- Wide use since many years

Josephson effect predicted in 1962, junction first made in 1963, first squid in 1964

Type I superconductors

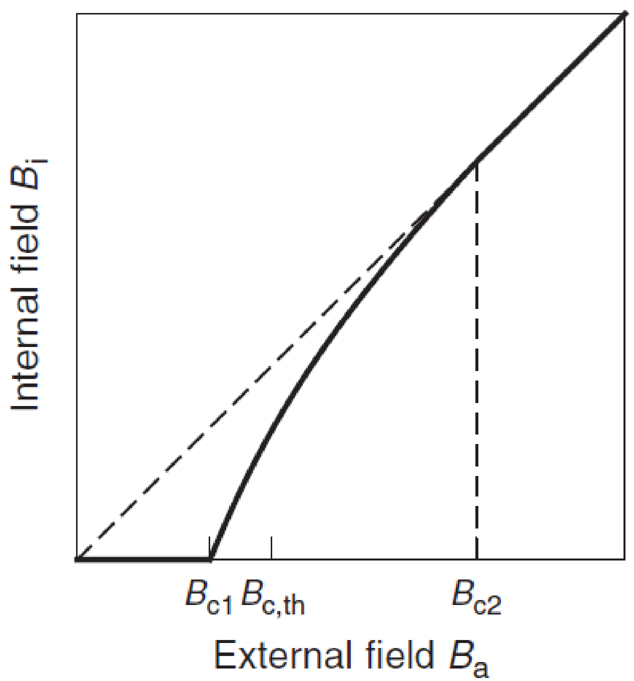
- Most pure metals (Pb, Hg, In, Al) are type I superconductors
- They are ideal diamagnets
- In external magnetic field superconductivity breaks down above critical field B_c
- See diagram of field inside superconductor B_i as function of external field B_a and the negative magnetization of the superconductor (for T=0)

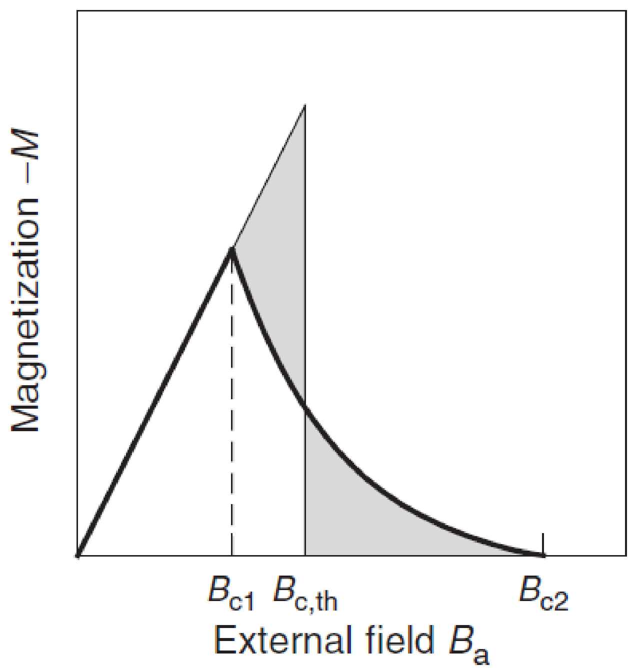




Type II superconductors

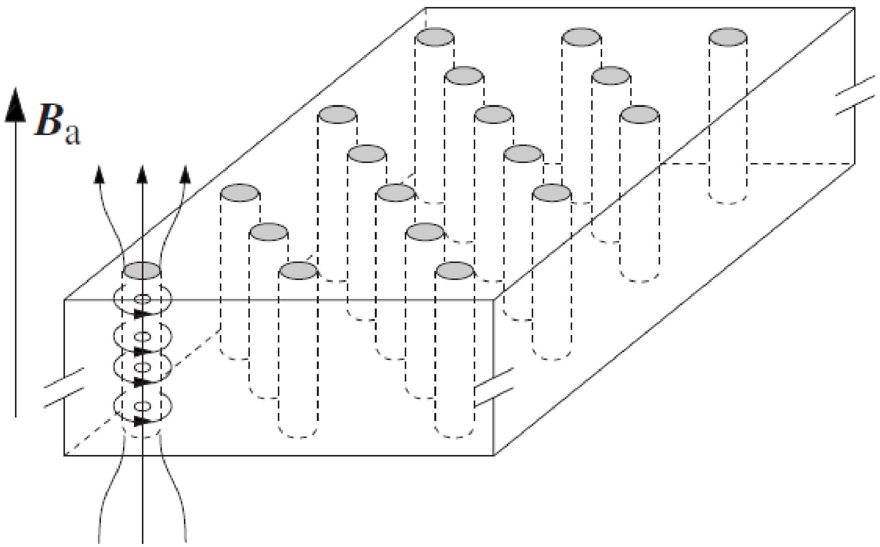
- Transition metals, metallic glasses, novel high-T_c superconductors, are type II superconductors
- Same behaviour below lower critical field: $0<B<B_{c1}$
- At higher field normal conducting islands are formed inside superconductive volume and field penetrates partially: mixed state for $B_{c1}<B<B_{c2}$
- See diagram of field inside superconductor B_i as function of external field B_a and the negative magnetization of the superconductor (for T=0)

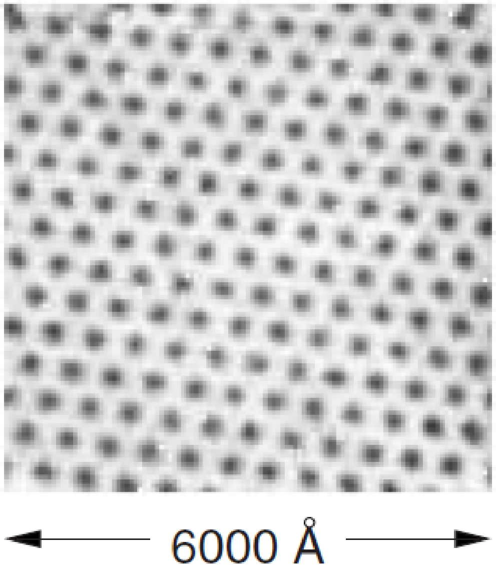




Type II superconductors

- Type II superconductors more practical for high field applications (magnet coils): high B_{C2} and T_C
- Flux lines regularly arranged in perfect crystals (Abrikosov lattice, see graph) as this minimizes energy through equal separation
- In practice sample impurities can pin flux lines





Abrikosov lattice sketch (left) and observation (right) in $NbSe_2$ at 1.8K and B=1T, using STM

High T_C superconductors

- For elements highest T_C=9.5K (Nb)
- For alloys highest T_C=23.2K (Nb_3Ge)

Since 1986:

- $La_{1.85}Sr_{0.15}CuO_4$: T_C=36K
- $YBa_2Cu_3O_7$: T_C=90K
- $Tl_2Ba_2Ca_2Cu_3O_{10}$: T_C=120K

High T_C superconductors

- Materials difficult to make
- Cannot transport large currents
- Intriguing as mechanism is still not clear

High T_C superconductors

- Crystal structure of CuO_2 layers spaced apart by other atoms
- Anisotropic behaviour – superconductivity mainly along planes
- High T_C indicates strong paring interaction
- Small size of Cooper pairs
- April 2008: LaOFeAs (first not cuprate high T_C superconductor) with T_C up to 50K; very little theoretical understanding